

CLAIMS

What is claimed is:

1. A method of inverting a 4x4 source matrix, the method comprising:
dividing the source matrix into four 2x2 sub-matrices A , B , C and D ;
calculating a plurality of sub-matrix products from the sub-matrices;
calculating a determinant of the source matrix dS to form a matrix determinant residue rd of the source matrix as $rd=1/dS$;
forming a partial, inverse sub-matrix of each sub-matrix using one or more of the matrix products and a determinant of each sub-matrix; and
calculating an inverse of each sub-matrix iA , iB , iC , and iD , utilizing each partial, inverse sub-matrix and determinant residue rd , such that an inverse of the source matrix iS is formed.

2. The method of claim 1, wherein dividing the source matrix S into the four 2x2 sub-matrices A , B , C and D is performed according to the following rule:

$$S = \begin{pmatrix} A & B \\ C & D \end{pmatrix}$$

to enable storage of each sub-matrix within a pair of SIMD registers.

3. The method of claim 1, wherein calculating the plurality of sub-matrix products further comprises:

calculating an intermediate sub-matrix product for each sub-matrix by computing the following matrix equations:

$$\tilde{D}C = \text{adj}(D) \bullet C$$

$$\tilde{A}B = \text{adj}(A) \bullet B$$

wherein the adj function refers to an adjoint matrix operation and the dot symbol \bullet refers to a matrix multiplication operation; and

calculating a final sub-matrix product for each of the intermediate sub-matrix products by computing the following equations:

$$\begin{aligned} B\tilde{D}C &= B \bullet \tilde{D}C \\ D\tilde{B}A &= D \bullet \text{adj}(\tilde{A}B) \\ A\tilde{C}D &= A \bullet \text{adj}(\tilde{D}C) \\ C\tilde{A}B &= C \bullet \tilde{A}B. \end{aligned}$$

4. The method of claim 1, wherein calculating the matrix determinant residue further comprises:

computing a determinant of each sub-matrix dA , dB , dC and dD ;
calculating a trace value by computing a following equation:

$$t = \text{trace}(\tilde{A}B \bullet \tilde{D}C);$$

wherein a dot symbol \bullet refers to a matrix multiplication operation; and

calculating a determinant of the source matrix dS by computing a following equation:

$$dS = dA * dD + dB * dC - t$$

wherein the symbol $*$ refers to a scalar multiplication operation.

5. The method of claim 1, wherein forming partial-inverse sub-matrices further comprises:

performing matrix scaling of a determinant of each sub-matrix as $D*dA$, $C*dB$, $B*dC$ and $A*dD$; and

computing a partial inverse for each sub-matrix according to the following matrix scaling equations:

$$\begin{aligned} pA &= A * dD - B\tilde{D}C \\ pB &= C * dB - D\tilde{B}A \\ pC &= B * dC - A\tilde{C}D \\ pD &= D * dA - C\tilde{A}B, \end{aligned}$$

wherein pA , pB , pC , and pD reference partial, inverse sub-matrices, and the symbol $*$ refers to a matrix scaling by a scalar operation.

6. The method of claim 1, wherein calculating an inverse of each sub-matrix further comprises:

calculating an adjoint value of each partial, inverse sub-matrix pA , pB , pC , and pD , according to the following rules:

$$iA = \text{adj}(pA),$$

$$iB = \text{adj}(pB),$$

$$iC = \text{adj}(pC),$$

$$iD = \text{adj}(pD),$$

wherein the $\text{adj}()$ function refers to the adjoint matrix operation;

calculating a final sub-matrix inverse value according to the following equations:

$$iA = iA * rd$$

$$iB = iB * rd$$

$$iC = iC * rd$$

$$iD = iD * rd,$$

wherein the symbol $*$ refers to a matrix scaling by a scalar operation; and forming the inverse source matrix iS according to the following rule:

$$iS = \begin{pmatrix} iA & iB \\ iC & iD \end{pmatrix}.$$

7. A method comprising:

dividing a source matrix into four 2×2 sub-matrices, A , B , C and D ;

calculating one or more intermediate sub-matrix products from one or more of the sub-matrices;

calculating a determinant of the source matrix to form a determinant residue rd utilizing the intermediate sub-matrix products;

scaling a determinant of each sub-matrix and the intermediate sub-matrix products using determinant residue rd to form final sub-matrix products;

forming a partial inverse sub-matrix pA , pB , pC and pD for each sub-matrix using the scaled sub-matrix determinants and the final sub-matrix products; and

calculating an inverse of each sub-matrix iA , iB , iC and iD , utilizing each partial inverse sub-matrix to form an inverse source matrix iS .

8. The method of claim 7, wherein calculating the matrix determinant residue further comprises:

computing a determinant of each sub-matrix dA , dB , dC and dD ;

calculating a trace value by computing a following equation:

$$t = \text{trace}(\tilde{A}B \bullet \tilde{D}C);$$

wherein a dot symbol \bullet refers to a matrix multiplication operation;

calculating a determinant of the source matrix dS by computing a following equation:

$$dS = dA * dD + dB * dC - t$$

wherein the symbol $*$ refers to a scalar multiplication operation; and

calculating the determinant residue rd according to the following rule:

$$rd = 1/dS.$$

9. The method of claim 7, wherein scaling by the determinant residue further comprises:

multiplying each determinant by the determinant residue rd according to the following rules:

$$dA = dA * rd$$

$$dB = dB * rd$$

$$dC = dC * rd$$

$$dD = dD * rd;$$

multiplying each intermediate sub-matrix product $\tilde{A}B$ and $\tilde{D}C$ by the determinant residue rd , according to the following equations:

$$\tilde{D}C = \tilde{D}C * rd$$

$$\tilde{A}B = \tilde{A}B * rd; \text{ and}$$

calculating a final sub-matrix product for each of the intermediate matrix products by computing the following equations:

$$B\tilde{D}C = B \bullet \tilde{D}C$$

$$D\tilde{B}A = D \bullet \text{adj}(\tilde{A}B)$$

$$A\tilde{C}D = A \bullet \text{adj}(\tilde{D}C)$$

$$C\tilde{A}B = C \bullet \tilde{A}B.$$

10. The method of claim 7, wherein calculating an inverse of each sub-matrix further comprises:

generating an adjoint of each partial, inverse sub-matrix by computing the following equations:

$$iA = \text{adj}(pA)$$

$$iB = \text{adj}(pB)$$

$$iC = \text{adj}(pC)$$

$$iD = \text{adj}(pD); \text{ and}$$

forming the inverse source matrix iS according to the following rule:

$$iS = \begin{pmatrix} iA & iB \\ iC & iD \end{pmatrix}.$$

11. A computer readable storage medium including program instructions that direct a computer to function in a specified manner when executed by a processor, the program instructions comprising:

dividing the source matrix into four 2×2 sub-matrices A, B, C and D;

calculating a plurality of sub-matrix products from the sub-matrices;

calculating a determinant of the source matrix dS to form a matrix determinant residue rd of the source matrix as $rd=1/dS$;

forming a partial, inverse sub-matrix of each sub-matrix using one or more of the matrix products and a determinant of each sub-matrix; and

calculating an inverse of each sub-matrix iA , iB , iC , and iD , utilizing each partial, inverse sub-matrix and determinant residue rd , such that an inverse of the source matrix iS is formed.

12. The computer readable storage medium of claim 11, wherein dividing the source matrix S into the four 2×2 sub-matrices A, B, C and D is performed according to the following rule:

$$S = \begin{pmatrix} A & B \\ C & D \end{pmatrix}$$

to enable storage of each sub-matrix within a pair of SIMD registers.

13. The computer readable storage medium of claim 11, wherein calculating the plurality of sub-matrix products further comprises:

calculating an intermediate sub-matrix product for each sub-matrix by computing the following matrix equations:

$$\tilde{D}C = \text{adj}(\tilde{D}) \bullet C$$

$$\tilde{A}B = \text{adj}(A) \bullet B$$

wherein the $\text{adj}()$ function refers to an adjoint matrix operation and the dot symbol \bullet refers to a matrix multiplication operation; and

calculating a final sub-matrix product for each of the intermediate sub-matrix products by computing the following equations:

$$B\tilde{D}C = B \bullet \tilde{D}C$$

$$D\tilde{B}A = D \bullet \text{adj}(\tilde{A}B)$$

$$A\tilde{C}D = A \bullet \text{adj}(\tilde{D}C)$$

$$C\tilde{A}B = C \bullet \tilde{A}B.$$

14. The computer readable storage medium of claim 11, wherein calculating the matrix determinant residue further comprises:

computing a determinant of each sub-matrix dA , dB , dC and dD ;

calculating a trace value by computing a following equation:

$$t = \text{trace}(\tilde{A}B \bullet \tilde{D}C);$$

wherein a dot symbol \bullet refers to a matrix multiplication operation; and

calculating a determinant of the source matrix dS by computing a following equation:

$$dS = dA * dD + dB * dC - t$$

wherein the symbol $*$ refers to a scalar multiplication operation.

15. The computer readable storage medium of claim 11, wherein forming partial-inverse sub-matrices further comprises:

performing matrix scaling of a determinant of each sub-matrix as D^*dA , C^*dB , B^*dC and A^*dD ; and

computing a partial inverse for each sub-matrix according to the following matrix scaling equations:

$$pA = A * dD - \tilde{B}DC$$

$$pB = C * dB - \tilde{D}BA$$

$$pC = B * dC - \tilde{A}CD$$

$$pD = D * dA - \tilde{C}AB,$$

wherein pA , pB , pC , and pD reference partial, inverse sub-matrices, and the symbol * refers to a matrix scaling by a scalar operation.

16. The computer readable storage medium of claim 11, wherein calculating an inverse of each sub-matrix further comprises:

calculating an adjoint value of each partial, inverse sub-matrix pA , pB , pC , and pD , according to the following rules:

$$iA = \text{adj}(pA),$$

$$iB = \text{adj}(pB),$$

$$iC = \text{adj}(pC),$$

$$iD = \text{adj}(pD),$$

wherein the $\text{adj}()$ function refers to the adjoint matrix operation;

calculating a final sub-matrix inverse value according to the following equations:

$$iA = iA * rd$$

$$iB = iB * rd$$

$$iC = iC * rd$$

$$iD = iD * rd,$$

wherein the symbol * refers to a matrix scaling by a scalar operation; and

forming the inverse source matrix iS according to the following rule:

$$iS = \begin{pmatrix} iA & iB \\ iC & iD \end{pmatrix}.$$

17. The computer readable storage medium including program instructions that direct a computer to function in a specified manner when executed by a processor, the program instructions comprising:

dividing a source matrix into four 2x2 sub-matrices, A , B , C and D ;

calculating one or more intermediate sub-matrix products from one or more of the sub-matrices;

calculating a determinant of the source matrix dS to form a determinant residue rd of the source matrix utilizing the intermediate sub-matrix products and the sub-matrix determinants;

scaling a determinant of each sub-matrix and the intermediate sub-matrix products using determinant residue rd to form final sub-matrix products;

forming a partial inverse sub-matrix pA , pB , pC and pD for each sub-matrix using the scaled sub-matrix determinants and the final sub-matrix products; and

calculating an inverse of each sub-matrix iA , iB , iC and iD , utilizing each partial inverse sub-matrix to form an inverse source matrix iS .

18. The computer readable storage medium of claim 17, wherein calculating the matrix determinant residue further comprises:

computing a determinant of each sub-matrix dA , dB , dC and dD ;

calculating a trace value by computing a following equation:

$$t = \text{trace}(\tilde{A}B \bullet \tilde{D}C);$$

wherein a dot symbol \bullet refers to a matrix multiplication operation;

calculating a determinant of the source matrix dS by computing a following equation:

$$dS = dA * dD + dB * dC - t$$

wherein the symbol $*$ refers to a scalar multiplication operation; and

calculating the determinant residue rd according to the following rule:

$$rd = 1/dS.$$

19. The computer readable storage medium of claim 17, wherein scaling by the determinant residue further comprises:

multipling each determinant by the determinant residue rd according to the following rules:

$$dA = dA * rd$$

$$dB = dB * rd$$

$$dC = dC * rd$$

$$dD = dD * rd;$$

multipling each intermediate sub-matrix product by the determinant residue rd , according to the following equations:

$$\tilde{DC} = \tilde{DC} * rd$$

$$\tilde{AB} = \tilde{AB} * rd; \text{ and}$$

calculating a final sub-matrix product for each of the intermediate matrix products by computing the following equations:

$$B\tilde{DC} = B \bullet \tilde{DC}$$

$$D\tilde{BA} = D \bullet \text{adj}(\tilde{AB})$$

$$A\tilde{CD} = A \bullet \text{adj}(\tilde{DC})$$

$$C\tilde{AB} = C \bullet \tilde{AB}.$$

20. The computer readable storage medium of claim 17, wherein calculating an inverse of each sub-matrix further comprises:

generating an adjoint of each partial, inverse sub-matrix by computing the following equations:

$$iA = \text{adj}(pA)$$

$$iB = \text{adj}(pB)$$

$$iC = \text{adj}(pC)$$

$$iD = \text{adj}(pD); \text{ and}$$

forming the inverse source matrix iS according to the following rule:

$$iS = \begin{pmatrix} iA & iB \\ iC & iD \end{pmatrix}.$$

21. An apparatus, comprising:

- a processor having circuitry to execute instructions;
- a plurality of SIMD data storage devices coupled to the processor, the SIMD data storage registers to pairs of floating point vectors during matrix calculation;
- a storage device coupled to the processor, having sequences of instructions stored therein, which when executed by the processor cause the processor to:
 - divide the source matrix into four 2x2 sub-matrices A , B , C and D ;
 - calculate a plurality of sub-matrix products from the sub-matrices;
 - calculate a determinant of the source matrix dS to form a determinant residue rd of the source matrix as $rd=1/dS$;
 - form a partial, inverse sub-matrix of each sub-matrix using one or more of the matrix products and the determinant of each sub-matrix; and
 - calculate an inverse of each sub-matrix iA , iB , iC , and iD , utilizing each partial, inverse sub-matrix and determinant residue rd , such that an inverse of the source matrix iS is formed.

22. The apparatus of claim 21, wherein the instruction to calculate the plurality of sub-matrix products further causes the processor to:

calculate an intermediate sub-matrix product for each sub-matrix by computing the following matrix equations:

$$\tilde{D}C = \text{adj}(\tilde{D}) \bullet C$$

$$\tilde{A}B = \text{adj}(A) \bullet B$$

wherein the $\text{adj}()$ function refers to an adjoint matrix operation and the dot symbol \bullet refers to a matrix multiplication operation; and

calculate a final sub-matrix product for each of the intermediate sub-matrix products by computing the following equations:

$$B\tilde{D}C = B \bullet \tilde{D}C$$

$$D\tilde{B}A = D \bullet \text{adj}(\tilde{A}B)$$

$$A\tilde{C}D = A \bullet \text{adj}(\tilde{D}C)$$

$$C\tilde{A}B = C \bullet \tilde{A}B.$$

23. The apparatus of claim 21, wherein the instruction to calculate the matrix determinant residue further causes the processor to:

compute a determinant of each sub-matrix dA , dB , dC and dD ;
calculate a trace value by computing a following equation:

$$t = \text{trace}(\tilde{A}B \bullet \tilde{D}C);$$

wherein a dot symbol \bullet refers to a matrix multiplication operation; and

calculate a determinant of the source matrix dS by computing a following equation:

$$dS = dA * dD + dB * dC - t$$

wherein the symbol $*$ refers to a scalar multiplication operation.

24. The apparatus of claim 21, wherein the instruction to perform matrix scaling further causes the processor to:

perform matrix scaling of a determinant of each sub-matrix as $D*dA$, $C*dB$, $B*dC$ and $A*DdD$;

compute a partial inverse for each sub-matrix according to the following matrix scaling equations:

$$pA = A * dD - B\tilde{D}C$$

$$pB = C * dB - D\tilde{B}A$$

$$pC = B * dC - A\tilde{C}D$$

$$pD = D * dA - C\tilde{A}B,$$

wherein pA , pB , pC , and pD reference partial, inverse sub-matrices and the symbol $*$ refers to a matrix scaling by a scalar operation.

25. The apparatus of claim 21, wherein the instruction to calculate an inverse of each sub-matrix further causes the processor to:

calculate an adjoint value of each partial, inverse sub-matrix pA , pB , pC , and pD , according to the following rules:

$$iA = \text{adj}(pA),$$

$$iB = \text{adj}(pB),$$

$$iC = \text{adj}(pC),$$

$$iD = \text{adj}(pD),$$

wherein the $\text{adj}()$ function refers to the adjoint matrix operation;

calculate a final sub-matrix inverse value according to the following equations:

$$iA = iA * rd$$

$$iB = iB * rd$$

$$iC = iC * rd$$

$$iD = iD * rd,$$

wherein the symbol * refers to a matrix scaling by a scalar operation; and form the inverse source matrix iS according to the following rule:

$$iS = \begin{pmatrix} iA & iB \\ iC & iD \end{pmatrix}.$$

26. An apparatus, comprising:

a processor having circuitry to execute instructions;

a plurality of SIMD data storage devices coupled to the processor, the SIMD data storage registers to pairs of floating point vectors during matrix calculation;

a storage device coupled to the processor, having sequences of instructions stored therein, which when executed by the processor cause the processor to:

divide a source matrix into four 2x2 sub-matrices, A , B , C and D ;

calculate one or more intermediate sub-matrix products from each of the sub-matrices,

calculate a source matrix dS to form a determinant residue rd utilizing the intermediate sub-matrix products,

scale a determinant of each sub-matrix and the intermediate sub-matrix products using determinant residue rd to form final sub-matrix products,

form a partial inverse sub-matrix pA , pB , pC and pD for each sub-matrix using the scaled sub-matrix determinants and the final sub-matrix products, and

calculate an inverse of each sub-matrix iA , iB , iC and iD , utilizing each partial inverse sub-matrix to form an inverse source matrix iS .

27. The apparatus of claim 26, wherein the instruction to calculate the source matrix determinant residue further causes the processor to:

compute a determinant of each sub-matrix dA , dB , dC and dD ;

calculate a trace value by computing a following equation:

$$t = \text{trace}(\tilde{A}B \bullet \tilde{D}C)$$

wherein a dot symbol \bullet refers to a matrix multiplication operation;

calculate a determinant of the source matrix dS by computing a following equation:

$$dS = dA * dD + dB * dC - t$$

wherein the symbol $*$ refers to a scalar multiplication operation; and

calculate the determinant residue rd according to the following rule:

$$rd = 1/dS.$$

28. The apparatus of claim 26, wherein the instruction to scale by the determinant residue further causes the processor to:

multiply each determinant by the determinant residue rd according to the following rules:

$$dA = dA * rd$$

$$dB = dB * rd$$

$$dC = dC * rd$$

$$dD = dD * rd;$$

multiply each intermediate sub-matrix product $\tilde{A}B$ and $\tilde{D}C$ by the determinant residue rd , according to the following equations:

$$\tilde{D}C = \tilde{D}C * rd$$

$$\tilde{A}B = \tilde{A}B * rd; \text{ and}$$

calculate a final sub-matrix product for each of the intermediate matrix products by computing the following equations:

$$B\tilde{D}C = B \bullet \tilde{D}C$$

$$D\tilde{B}A = D \bullet \text{adj}(\tilde{A}B)$$

$$A\tilde{C}D = A \bullet \text{adj}(\tilde{D}C)$$

$$C\tilde{A}B = C \bullet \tilde{A}B.$$

29. The apparatus of claim 26, wherein the instruction to calculate an inverse of each sub-matrix further causes the processor to:

generate an adjoint of each partial, inverse sub-matrix by computing the following equations:

$$iA = \text{adj}(pA)$$

$$iB = \text{adj}(pB)$$

$$iC = \text{adj}(pC)$$

$$iD = \text{adj}(pD); \text{ and}$$

form the inverse source matrix iS according to the following rule:

$$iS = \begin{pmatrix} iA & iB \\ iC & iD \end{pmatrix}.$$